Direct measurement of micromotion speed in a linear quadrupole trap

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We demonstrate a simple method to directly measure the micromotion speed and amplitude of ions far away from the nodal line of the linear quadrupole trap using the cross-correlation technique. For the ions very close to the trap nodal line, the micromotion speed and amplitude of ions can also be deduced through linear fitting. This work gives us a direct picture to the ions’ micromotion modes at different displacements in the linear trap. With this work, an absolute measurement of the second-order Doppler effect in the research of atomic clocks based on large number of ions becomes possible. © 2010 American Institute of Physics. [doi:10.1063/1.3457904]

I. INTRODUCTION

Trapping of large ion clouds in linear Paul traps and laser cooling them to crystallization temperatures have many applications on the studying of ions collision processes,1,2 quantum information processing,3,4 phase transition and crystal structure,5–8 and high-accuracy spectroscopy and frequency standards.9,10 In a large ion cloud, ion micromotion, the periodic ion motion that is synchronous with the trap driving electric field can have many negative effects. These effects include the increasing of the cooling temperature,11–13 the alterations of the atomic transition line shapes,14 second-order Doppler shift of the atomic transition lines,15,16 and limited confinement time in the absence of cooling light.17 For single ion or a single line of ions trapped in linear quadrupole trap, the micromotion can be minimized to almost zero by shifting the ions’ position to the nodal line of the trap by using dc compensation electric field. However, for large ion clouds or ion crystals trapped in a linear quadrupole trap, the ions are pushed away by Coulomb interactions. As a consequence, not all ions can occupy the nodal line of the trap, which leads to the unavoidable micromotion associated with the rf driving field.18 It was also pointed out by Prestage et al.19 that, for larger ion clouds, most of the motion energy is stored in the micromotion, which is necessary to generate the force to balance the space-charge repulsion.

Even though the micromotion amplitude can be estimated theoretically,14,20 considering the trap field always be distorted in experiment (by trap’s geometrical design, metal as well as dielectric elements inside the vacuum chamber, and so on.), the calculated result cannot give the real picture of the ion’s micromotion mode. For many high precision ion trap applications, the quantitative micromotion amplitude and speed is needed to evaluate or reduce the negative effects of large ion cloud’s or crystal’s micromotion. For example, to precisely compute the second-order Doppler frequency offset of an atomic clock based on a large number of ions, we must know the micromotion speed throughout the ion trap accurately.19 Consequently, besides theoretical calculation, it is still necessary to directly measure the real micromotion amplitude and speed of the ions at different positions in a linear trap. On the other hand, besides the negative effects of the micromotion, the ion with rf-driven micromotion is also a self-sustained oscillator (SSO).21 This “classical” dynamic oscillation for trapped ions, atoms, and molecules becomes a fascinating subject in recent years, with many potential applications.22,23 For these SSO systems, the oscillation amplitude or speed are important parameters. Consequently, direct measurement of the micromotion amplitude and speed throughout the trapping region becomes necessary indeed. The direct measurement of the micromotion speed of different layers of a large ion crystal is difficult due to the very small distance between different layers and the fluorescence background generated by the ions on other layers. As an alternative method, we trap a single layer of 138Ba+ ion crystals and displace them to different positions of the trap by the dc compensation electric field. The micromotion amplitude and speed at these positions are then measured directly. This method is valid since in the mean field approach the Coulomb repulsion field can be seen as a static field.24

Several methods can be used to detect and compensate the micromotion of the trapped ions, such as position compensation, linewidth compensation, micromotion sideband compensation, and cross-correlation compensation.14 Among them, the cross-correlation technique is the most sensitive method for ions whose cooling transition linewidth is far larger than the rf trapping frequency (weak binding limit), as in the case of Barium ions. Reference 14 uses this technique to minimize the micromotion of trapped ions in the trap axes and estimates the fractional second-order Doppler shift through theoretical fitting. Using this technique, we demonstrate a simple (but largely overlooked) method which can be used to directly measure the micromotion speed of ions far away from the nodal line of the ion trap. For ions very close to the trap nodal line, the micromotion speed and amplitude of the ions can also be deduced through a linear fitting. To the best of our knowledge, this is the first time to directly measure the micromotion speed of ions far away from the nodal line of linear quadrupole trap. The results of this work can be useful for the research of atomic clocks based on large number of ions,10,19,25 laser cooling of ion crystals,26,27 and the “classical” dynamic oscillation for trapped ions.22,23 For example, with this technique, a direct picture of the ions’ micromotion modes throughout the trap region can be given.
which makes an absolute measurement of the second-order Doppler effect in the research of atomic clocks possible; With this technique, extremely small force (for example the dc compensation force) applied on the trapped ions can also be detected. Recently, similar technique has been used to detect the YoctoNewton force on the trapped ions in a Penning trap.28

II. MEASUREMENT PRINCIPLE

The driven motion of the ions with the rf driving frequency \( \Omega \) causes the fluorescence rate to be modulated at the same frequency via first-order Doppler shift. In the low intensity and \( \Omega \ll \Gamma \) limit (\( \Gamma \) is the atomic transition linewidth), the fluorescence rate of the ions in the rf-driven motion can be written as

\[
\kappa = \frac{(\Gamma/2)^2}{(\Gamma/2)^2 + (\omega_0 - \omega + k \cdot \vec{v}(t))^2},
\]

where \( \kappa_{0} \) is the maximum fluorescence rate at resonance, \( \omega_0 \) is the ion transition frequency, \( \omega \) is the frequency of the probe light, \( k \) is the wave vector of the probe light, and \( \vec{v}(t) \) is the velocity vector of the micromotion. In this work, we only consider the micromotion in one direction. In this case, Eq. (1) can be written as

\[
\kappa = \frac{(\Gamma/2)^2}{(\Gamma/2)^2 + (\omega_0 - \omega + \vec{k} \cdot \vec{v}(t))^2},
\]

where \( \kappa \) is the maximum speed of the rf driving micromotion, \( \phi \) is a phase determined by the initial position of the ion with respect to the driving field, \( c \) is the speed of light, and \( \delta_0 = \omega_0 - \omega \) is the frequency detuning of the probe light. When the micromotion speed \( u = 0 \), there is no modulation on the fluorescence signal, as shown in curve (i) of Fig. 1. When \( 0 < u < \delta_0/\omega \), which is the same case as that discussed in Ref. 14, the fluorescence signal will be modulated with a period \( T = 2\pi/\Omega \) (in our experiment, the rf frequency \( \Omega / 2\pi = 1.96 \) MHz, corresponding to \( T = 510.2 \) ns), and there is one peak on the fluorescence signal in each period, as shown in curve (ii) of Fig. 1. When \( u = \delta_0/\omega \), the fluorescence rate will reach a maximum at the time of \( t = (2n\pi - \phi)/\Omega \), with \( n = 1, 2, 3, \ldots \), and there is a small flat top on the fluorescence signal, as shown in curve (iii) of Fig. 1. When \( u > \delta_0/\omega \), there will be two peaks during each period of the fluorescence modulation signal, and the peaks appear at the time of \( t = (2n\pi \pm \arccos(\delta_0/\omega) - \phi)/\Omega \), with \( n = 1, 2, 3, \ldots \), as shown in curves (iv) and (v) of Fig. 1.

There are five characteristic times in curves (iv) and (v), and we can use curve (v) as an example. Points A and B represent the times when the ions’ instantaneous micromotion speeds are zero. All curves [including curves (i), (ii), and (iii)] will pass through these two points and the time difference between points A and B is exactly \( t_{AB} = T/2 \); symmetric point C represents the time when the ions have their maximum speeds toward the probe light; points D and E represent the time when the ions are resonant with the probe light. From the time difference of points D and E, we can deduce the micromotion speed of ions as

\[
u = \frac{\delta_0 c}{\omega \cos(\pi \delta_{DE}/T)}.
\]

In the above formula, the exact value of the probe detuning \( \delta_0 \) is difficult to measure directly, but we can precisely change its relative detuning \( \delta_{r} \), where \( \delta_{r} = \delta_0 + \delta_{r} \). Then Eq. (3) can be rewritten as

\[
\cos \left( \frac{\pi \delta_{DE}}{T} \right) = \frac{c \delta_{r} + c \delta_{0}}{\omega u}.
\]

By measuring \( \delta_{DE} \) under different relative detuning \( \delta_{r} \) and least-square-fitting the results of \( \cos(\pi \delta_{DE}/T) \), we can determine the absolute value of the reference probe detuning \( \delta_{0} \) from the ratio between the intercept point and the slope.

III. EXPERIMENTAL PROCEDURES

The experimental setup is shown in Fig. 2. The design of the linear quadrupole trap that we used is similar to that of
Ref. 29. All of the even numbered electrodes [see Fig. 2(a)] are applied an rf voltage of the form $V_c \cos(\Omega t)$. A typical rf driving frequency is $\Omega = 2\pi \times 1.96$ MHz, with amplitude $V_c = 100$ V. All of the odd numbered electrodes are ac grounded. On the eight end electrodes (electrodes 1–4 and 9–12), a positive dc voltage ($U_z = 18$ V) is added to the rf voltage (or ground) through LC coupling to provide the axial confinement. These eight end electrodes serve as the endcaps. To compensate and change the dc field in the trap center, we add two separate dc voltages $U_5$ and $U_6$, as shown in Fig. 2(b). The length of the center electrodes and end-caps are 25 mm and 17 mm, respectively. The diameters of all electrodes are $2R = 8.000$ mm, and the distance between two diagonal electrodes inner surfaces is $2r_0 = 7.000$ mm. The ratio between these two numbers is 1.143. This value is within the range of 1.130–1.145,30–32 which ensures an approximate quadrupole field in the trap center and reduces the secular motion heating effect.33

Barium ions are trapped in the experiment. The cooling laser at 493 nm and repumping laser at 650 nm are coupled into a single mode fiber and directed into the trap along the $-\hat{z}$ direction. The diameters of these two beams are 2 mm inside the trap. To detect the micromotion of ions, another set of probe and repumping beams are split from the cooling and repumping beams using a polarization beam splitter and directed into the trap center along the $\hat{x} - \hat{z}$ direction. With this arrangement we can detect the micromotion of ions along $\hat{x}$ direction. The micromotion of ions along $\hat{z}$ direction is very small and cannot be detected in our experiment. The ions are imaged onto a charge coupled device (CCD) camera (Olympus CC-12) with a $3 \times 3$ imaging system. The whole system is mounted on a three-axis translation stage with a motional resolution of 0.1 $\mu$m. In addition, the fluorescence of the ions is detected by a photomultiplier tube (PMT) photon counting head (Hamamatsu H8259). The PMT output is used to produce the START signal for a time interval counter (Stanford Research Systems SR620). Pulse synchronized to the rf drive frequency serves as the STOP signal of the time interval counter. After accumulation of 20 000 photon counts, a histogram of time intervals can be built up, which gives the correlations (the cross-correlation signal) between the arrival of photons at the PMT and the phase of the rf field. This histogram represents the ions fluorescence rate at different time within one rf period, as described by Eq. (2).

During the experiment, we trap one line of about 40 $^{138}$Ba$^+$ ions. The ions are first cooled to crystallization temperature, as shown in Fig. 2(a). To measure the micromotion of ions, we switch off the cooling and repumping beams along the $\hat{z}$ direction, and switch on the probe and repumping beams along $\hat{x} - \hat{z}$ direction. The probe detuning $\delta_0$ is set to about the value of $\Gamma$, and its power (about 1 mW) is smaller than the saturation power of cooling transition. By adjusting the dc voltages (U5 and U6) applied on the electrodes 5 and 6, the ions can be moved to the nodal line of the linear trap, and their micromotion can be minimized. In our case, when $U_{50} = -310$ mV, $U_{60} = 396$ mV, the micromotion of the ions is minimized. Figure 3(a) shows the measured cross-correlation signal under this condition. The fluorescence rate is almost constant over the entire rf period, which means the micromotion is too small to be measured.

After moving the ions to the nodal line, an additional displacement voltage $\Delta U$ is subtracted from U5 and added to U6, such that $U_5 = -310 - \Delta U$ and $U_6 = 396 + \Delta U$. The units are in millivolt. This displacement voltage moves ions along the $\hat{y}$ direction. The displacement distance of the ions from the trap center can be measured by the imaging system together with the translation stage system. We first focus the image of ions onto the CCD camera. After displacing the ions from the trap nodal line, we then move the whole imaging system using the translation stage until the image appears at the same points on the CCD camera again. The displacement distance of the ions is the same as the translation stage’s moving distance. Figure 4 gives the measured relationship between the displacement of ions from the trap nodal line along $\hat{y}$ direction with the displacement voltage $\Delta U$. Through linear fitting, their relationship can be expressed as $d = 0.34\Delta U$, where the units of $d$ and $\Delta U$ are in micrometer and millivolt, respectively.

**IV. RESULTS AND DISCUSSION**

Figure 3 shows the cross-correlation signals when the ions are displaced along $\hat{y}$ direction at different displacement
voltages $\Delta U$. When $\Delta U = -20$ mV, as shown in Fig. 3(b), there is one peak on the cross-correlation signal in each period, which is the same case of curve (ii) of Fig. 1. When we further increase the displacement voltage to $\Delta U = -70, -130,$ and $-300$ mV [shown in Figs. 3(c)–3(e)], two peaks will appear within one rf period and the time difference between the two peaks will increase with the increasing displacement voltage. We also measure the cross-correlation signals when the ions are displaced along $-\hat{y}$ direction with positive displacement voltage $\Delta U$. The signals are similar to that of Fig. 3, except that there are $T/2$ time delay for each peak. This is because the RF driving field has a $\pi$ phase difference between $+\hat{y}$ region and $-\hat{y}$ region.

From the two-peak structure in Figs. 3(c)–3(e), we can then determine the micromotion speeds at different ions displacement positions. First we need to determine the reference probe detuning $\delta_0$. To do that, with $\Delta U = -300$ mV, we change the relative probe detuning $\delta$, and measure the corresponding time difference $t_{DE}$ of the two peaks of the cross-correlation signal. According to Eq. (4), we can plot the measured linear relationship between $\cos(t_{DE} \pi / T)$ and the relative probe detuning $\delta$, as shown in Fig. 5. Through a linear fitting, we can determine the reference probe frequency detuning $\delta_0/2\pi = 18.06$ MHz.

After that, since we have already measured the cross-correlation signals at different displacement voltages along $\hat{y}$ direction (Fig. 3), and have obtained the relationship between the displacement of ions from the trap nodal line along $\hat{y}$ direction and the displacement voltage $\Delta U$ (Fig. 4), based on Eq. (3), we can give the measured micromotion speed at different displacement along the $\hat{y}$ direction, as shown in Fig. 6. Because the micromotion of ions in $-\hat{y}$ region have a $\pi$ phase difference with that in $+\hat{y}$ region, for clarity, we set the micromotion speed of ions in the $-\hat{y}$ region as negative. As the micromotion of ions can be seen as a harmonic motion, the micromotion amplitude can then be calculated from $\alpha = u/\Omega$. As one can see, the micromotion speed and amplitude vary linearly with the ions’ displacement distance from the trap nodal line. Even though this method cannot directly measure the micromotion speed of ions when they are very close to the trap nodal line where $u < \delta c/\omega$, we can deduce the speed in these regions through linear fitting. Figure 6 gives a clear picture of the ion’s micromotion modes in the trap region, and it enables us to make an order of magnitude estimation for the negative effects caused by micromotion. For example, as can be seen in Fig. 6, at a mere distance of 50 $\mu$m away from the nodal line, the maximum micromotion speed reaches 23 m/s, corresponding to a fractional second-order Doppler shift of $2.9 \times 10^{-15}$ due to micromotion along $\hat{x}$ direction. This effect could limit the accuracy of optical atomic clocks based on large cloud of ions. One possible solution is to use a linear multipole trap,19,25,27 where the trapping electric field is much weaker over a large trapping volume.
As mentioned in Ref. 14, the micromotion amplitude for a quadrupole trap can be obtained as \(\alpha = qd/2\). The Mathieu parameter \(q\) is defined by
\[
q = \frac{2eA_2V_{\text{rf}}}{me^2r_0^2},
\] (5)
where \(m\) is the mass of the ion, \(e\) is the electron charge and \(A_2\) is the quadrupole coefficient which is determined by the ratio of rod radius to field radius \(R/r_0\).\(^{34,35}\) In our case, \(R/r_0 = 1.143\) corresponding to \(A_2 \approx 1.0026\).\(^{32}\) The Mathieu parameter can be calculated to be \(q = 0.0755\). The micromotion amplitude at \(d = 101\ \mu m\) is calculated to be \(\alpha = 3.81\ \mu m\), which is a little bit larger than the measured one \(\alpha = 3.64\ \mu m\) (in Fig. 6). The mismatch between theoretical calculation and experimental measurement is due to the imperfect quadrupole field in our linear trap, even though we have considered the quadrupole coefficient correction \((A_2)\) to the trap field. Beside the quadrupole coefficient \(A_2\) caused by the trap’s geometrical design mentioned above, the metal as well as dielectric elements (oven, electron gun, antenna, and mounting elements) inside the vacuum chamber also alter the rf trapping field. These problems will cause a shielding effect to the rf field. Here, the effective rf voltage can be approximately written as \(V_{\text{eff}} = \eta V_{\text{rf}}\), where \(\eta\) is the shielding factor, which cannot be calculated precisely. Normally, the shielding factor differs for the radial and axial directions of the linear trap. Hence, to obtain these shielding factors, one has to measure the secular frequencies of the ions in both the axial and radial directions. Then, by fitting the relationship between the secular frequency in radial direction and the rf voltage, one can obtain the radial shielding factor.\(^{36}\) This is a complicated process. Using the method we demonstrated here, the micromotion amplitude and speed can be measured directly. Further more, the shielding factor \(\eta\) can be deduced by comparing the measured micromotion amplitude with the calculated one.

V. CONCLUSION

In conclusion, we have demonstrated a comprehensive method to directly measure the ions’ micromotion speed and amplitude far away from the trap nodal line, using a cross-correlation technique. For ions very close to the trap nodal line of linear quadrupole trap, their micromotion speed and amplitude can also be deduced through linear fitting. This work gives a direct picture to the ions’ micromotion mode at different displacements in the linear quadrupole trap.


